On localization of acoustic waves

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(Nov. 14, 1998)

The paper presents a discussion on localization of acoustic waves. Some important questions about wave localization are addressed. Particular attention is paid to acoustic localization in liquid media containing many air-filled bubbles. It is shown that an amazing collective behavior appears when localization occurs. The study sheds new light to the much discussed subject.

PACS numbers: 43.20., 71.55J, 03.40K. To appear in Chin. J. Phys. 1999

I. INTRODUCTION

When propagating through media containing many scatterers, waves will be scattered by each scatterer. The scattered waves will be scattered again by other scatterers. This process is repeated to establish an infinite recursive pattern of rescattering between scatterers, forming a multiple scattering process which causes the scattering characteristics of the scatterers to change. Multiple scattering of waves is responsible for a wide range of fascinating phenomena. This includes, on large scales, twinkling light in the evening sky, modulation of ambient sound at ocean surfaces [1], acoustic scintillation from turbulent flows [2] and fish schools [3]. On smaller scales, phenomena such as white paint, random laser [4], electron transport in impured solids [5] and photonic band gaps in periodic structures [6] are also explained in terms of multiple scattering. Under proper conditions, multiple scattering leads to a phenomenon termed *localization*, which has now become an everyday experience.

Wave localization is a ubiquitous phenomenon. It refers to situations in which transmitted waves in a scattering medium are trapped in space and will remain confined in the neighborhood of the initial site until dissipated. The concept of localization was conceived from the theory describing the disorder induced conductor-insulator transition in electronic systems [7]. Since its inception, wave localization has stimulated considerable interest among scientists from many areas of disciplinary. Several monographs have been devoted to the subject (e. g. Ref. [8]). Wave localization may be realized in a variety of situations. In disordered solids, electron localization is common. Localization has also been reported for microwaves in random scattering rods and spheres [9,10], and recently for light in a ground gallium-arsenide suspension in methanol [11]. Acoustic localization has also been studied both theoretically [12] and experimentally [13]. Research suggests that acoustic localization may be observed in bubbly liquids [12,14].

Despite the tremendous efforts, however, no deeper insight into localization has been documented in the literature. The general cognition remains that enhanced backscattering is a precursor to wave localization and that disorder is an essential ingredient of localization. Important questions such as how and when the localization occurs stay as an unsolved puzzle.

In a recent paper [14], we proposed to study wave localization phenomenon by investigating wave propagation in liquid media containing many air-filled bubbles. There are several advantages of studying sound in bubbly liquids. (1) The air-filled bubbles are strong acoustic scatterer. At low frequencies, about $ka \sim 0.0136$, it appears the resonant scattering and the scattering strength is greatly enhanced at resonance, making it an ideal system to study strong scattering. Here k is the acoustic wavenumber in water and a is the radius of the bubbles. (2) The scattering function of a single bubble has been well studied and has a simple form. The scattering function of a spherical bubble can be found in many textbooks [15], whereas the scattering function of a deformed bubble, such as an ellipsoidal bubble, has also been analytically derived recently [16]. It is shown that the scattering function of a single bubble has a simple isotropic resonant form, permitting many simplifications. Furthermore, perhaps more important, such an isotropic scattering feature remains valid even as the bubbles are subject to significant deformation [16]. (3) Each term in the scattering function has clear physical meaning, and can be modified according to the need. When the thermal exchange and viscosity effects are taken into account, absorption indeed shows up but in a way such that it can be turned off or adjusted. This allows to unequivocally isolate localization due to scattering from attenuation caused by absorption, making waves in bubbly water an ideal system for studying phenomena related to multiple scattering.

In this paper, we first discuss general aspects of wave localization in a 3D system. An intuitive picture is proposed to describe wave localization and is subsequently supported by numerical examples in acoustic propagation in bubbly liquids. Wave localization is a phase transition. A novel method is proposed to describe such a phase transition. It

is shown that when the localization occurs, all air-filled bubbles as a resonant scatterer prevail a surprising collective behavior.

II. GENERAL ASPECTS

Consider a plane wave normally incident upon a semi-infinite random medium. The transport equation for the total energy intensity I may be intuitively written as

$$\frac{dI}{dx} = -\alpha I, \tag{1}$$

where α represents decay along the path traversed. After penetrating into the random medium, the wave will be scattered by random inhomogeneities. As a result, the wave coherence starts to decrease, yielding the way to incoherence. Extinction of the coherent intensity I_C is described by

$$\frac{dI_C}{dx} = -\gamma I_C, \tag{2}$$

with the attenuation constant γ. Eqs. (1) and (2) lead to the exponential solutions

$$I(x) = I(0)e^{-\alpha x}, \quad \text{and} \quad I_C(x) = I(0)e^{-\gamma x}. \tag{3}$$

In deriving these equations, the boundary condition was used; it states that $I(0) = I_C(0)$ as no scattering has been incurred yet at the interface. According to energy conservation, the incoherent intensity I_D (diffusive) is thus

$$I_D(x) = I(x) - I_C(x). \tag{4}$$

When there is no absorption, the decay constant α is expected to vanish and the total intensity will then be constant along the propagation path. Then, the coherent energy gradually decreases due to random scattering and transforms to the diffusive energy, while the sum of the two forms of energy remains a constant. This scenario, however, changes when localization occurs. Even without absorption, the total intensity can be localized near the interface due to multiple scattering. When this happens, α does not vanish. The transport of the total intensity may be still described by Eq. (1), and the inverse of α would then refer to the localization length.

The above perceptual description may be illustrated by Fig. 1. Without or with little absorption, the energy propagation is anticipated to follow the behavior depicted in (a). When the localization comes in sight, the wave will be trapped within an e-folding distance from the penetration, as prescribed by (b). In the non-localization case, the diffusive intensity increases steadily as more and more scattering occurs, complying with the Milne diffusion [15]. In the localized state, the diffusion energy increases initially. It will be eventually stopped by the interference of multiple scattering waves. Issues may be raised with respect to whether this apprehended image is supported by actual situations. In the rest, we will inspect this problem by considering acoustic waves in bubbly water.

III. ACOUSTIC LOCALIZATION IN BUBBLY LIQUIDS

Consider sound emission from a bubble cloud. For simplicity, the shape of the cloud is taken as spherical. Such a model eliminates irrelevant edge effects, and is useful to separate phenomena pertinent to the discussion. Total N bubbles of the same radius a are randomly distributed within the cloud. The volume void fraction, the space occupied by bubbles per unit volume, is take as β. A monochromatic acoustic source is located at the center of the cloud. Adaptation of such a model for other geometries and situations is straightforward. The wave transmitted from the source propagates through the bubble layer, where multiple scattering incurs, and then it reaches a receiver located at some distance from the cloud. The multiple scattering in the bubbly layer is described by a set of self-consistent equations. The energy transmission can be solved numerically in a rigorous fashion [14].

A traditional way to study wave localization is to calculate the Green's function for the energy transport, leading to the Bethe-Salpeter equation. Under certain approximations, such as long time and large distance from the initial site, a solution to this equation can be obtained in the form of a diffusion formula in which a mean free path and diffusion coefficient can be defined and have been used as the basis for discussion in the literature. Alternatively, certain situations allow direct computation of the energy transport, without recourse to unnecessary approximations. In the case, information about wave propagation can be inferred straightway. Acoustic wave propagation is one of such situations. The propagation is calculated incorporating all multiple scattering by the self-consistent scheme [14]. In the following, we will inspect the features of acoustic localization from three aspects and then present a discussion.

A. Frequency response

First consider the frequency response. In Fig. 2, the transmission in arbitrary units is plotted against frequency in terms of ka for two different bubble sizes. Here k is the usual wavenumber, and the bubble void fraction is 10^{-3}. It is clearly suggested in the figure that a narrow region is opened in which the transmission is virtually forbidden. This inhibition gap ranges roughly from $ka = 0.015$ to 0.02. When the void fraction is reduced to about 10^{-5}, the localization disappears [17]. In order to show that the inhibition is not because of absorption, the situation with the absorption being turned off is also plotted in the dotted line for $a = 2$cm. The comparison between with and without absorption excludes the absorption as the cause for the propagation hindrance. In fact, when the absorption factor of a single bubble is increased, the localization will be degraded.

B. Distance variation

To unambiguously identify that the transmission inhibition region is indeed the localization range, it is proper to study the spatial dependence of energy propagation. Fig. 3(a) presents the total transmission and its coherent and diffusive parts, scaled by the geometry spreading factor r^2, as a function of propagation distance scaled by the radius of the bubble cloud. The solid, dotted and broken lines refer to total, diffusive and coherent intensity respectively. The bubble radius is 2 cm, whereas the frequency is taken as $ka = 0.171$, which lies in the localization regime. When plotting the data in the natural logarithm in Fig. 3(b), we found that nearly all the data rest on a straight line. In (b), the straight line refers to the fitted curve and the crosses refer to the numerical data. We find the slope for the line amounts to a constant $\alpha = -0.195$, which is found to be true for other bubble sizes as well. This suggests that the intensity varies with propagation distance r as

$$I \sim \frac{e^{-\alpha r/a}}{r^2}. \tag{5}$$

Therefore the transport equation for the total wave can be written as

$$\frac{dI}{dr} = -\left(\frac{\alpha}{a} + \frac{2}{r}\right) I, \tag{6}$$

which is an equivalence of Eq. (1) in the spherical geometry. The second term on the right hand side of Eq. (6) denotes the geometrical spreading effect. From Eq. (5), the localization length is computed to be $l = 5.13a$. At this length, $kl \approx 0.0877$; therefore the Ioffe-Regel criterion for localization [8] is satisfied, providing another verification. At this frequency we also calculated attenuation length due to thermal and viscosity absorption to be about $118a$, which is much larger than the observed localization length.

The same calculation has also been performed for other sizes of bubbles. We found that within the localization regime, the ratio between the localization length and the size of bubble is nearly a constant around 5, as long as the bubble radius is large enough, roughly bigger than 10μm. For too small bubble, the absorption due to thermal exchange and viscosity effects is significant. Then the transmission will be size dependent and therefore the scaling behavior disappears. These features do not appear for ka outside the localization regime. Fig. 3(a) draws remarkable analogy to the perception demonstrated in Fig. 1(b).

C. Collective behavior

Upon incidence, each air-bubble acts as a secondary pulsating point source. The radiated wave from the i-th bubble ($i = 1, 2, 3, ..., N$) can be written as $A_i G_0(\vec{r} - \vec{r}_i)$, where $G_0(\vec{r} - \vec{r}_i)$ is the usual 3D Green's function and $\vec{r}_i$ denotes the positions of the bubble. The complex coefficient A_i refers to the effective strength of the secondary source, and is computed incorporating all multiple scattering effects. The total wave at any space point is the addition of the direct wave from the transmitting source and the radiated wave from all bubbles.

We express A_i as $|A_i| \exp(i\theta_i)$; the modulus A_i represents the strength, whereas θ_i the phase of the secondary source. We assign a unit vector $\vec{u}_i$, hereafter termed phase vector, to each phase θ_i, and these vectors are represented on an argand diagram in the $x-y$ plane. That is, the starting point of each phase vector is positioned at the center of individual scatterers with an angle with respect to the positive x-axis equal to the phase, $\vec{u}_i = \cos\theta_i \hat{x} + \sin\theta_i \hat{y}$. Letting the phase of the initiative emitting source be zero, numerical experiments are carried out to study the behavior of the phases of the bubbles and the spatial distribution of the acoustic energy. Figure 4 shows the argand diagrams of the

phase vectors, and the energy distribution for three frequencies in terms of ka for one arbitrary random configuration of bubbles.

We observe that for frequencies smaller than a certain value, there is no obvious order for the directions of the phase vectors, nor for the energy distribution. The phase vectors point to various directions, and no energy localization appears. The random behavior in the directions and energy distribution is attributed to the boundary effect of a finite number of the scatterers. In effect, as the wave is not localized, it can propagate through and is reflected by the asymmetric border; all bubbles can experience the effect via strong multiple scattering.

As the frequency increases, an order in the phase vectors and energy localization becomes evident. The case with $ka = 0.0164$ clearly shows that the energy is localized near the source, and amazingly, all bubbles oscillate completely in phase, but exactly out of phase with the transmitting source. Such collective behavior allows for efficient cancellation of incoming waves. The energy distribution decays exponentially, which sets the localization length. The localization behavior is independent of the outer boundary and always appears for sufficiently large β and N. When the frequency increases further, exceeding a certain amount, the in-phase order disappears. Meanwhile, the wave becomes non-localized again. This is illustrated by the case of $ka = 0.1$.

Further numerical investigation shows that the pattern depicted by Fig. 4 holds qualitatively true for other sizes of bubble (as long as the bubble is not too small; when the bubble is too small, the viscosity and thermal effects will dominate and the localization phenomenon will disappear. And the features are always valid for sufficiently large bubble void fraction.

D. On localization

Now a question immediately follows: What can we learn about wave localization from the above discussion? First, wave localization refers to trapping of the total wave energy, which may be indicated by the exponential decay of transmitted energies along the distance traveled by wave. However, the localization due to the scattering must be differentiated from the attenuation due to absorption. It is well-known that when absorption is present, energies will also decrease with the traveling distance. Second, when localization occurs, wave is trapped near the point of transmission. If a continuous wave is pumped into the system, there could be several conjectures. (1) The energy will be built up in the neighborhood of the transmission point until the amplitude is so large that localization fails. (2) When the localization occurs, no more energy can be pumped into the system. This scenario may be hinted by from Fig. 4 which shows that the anti-phase collective behavior allows for efficient cancellation of transmitting waves. This may be in analogy with situation for the electrical current in a conductor with resistance R that is connected to a battery with potential V. The electrical power is V^2/R. When added with sufficient amount of impurities, the conductor will become an insulator, i. e. $R \to \infty$. Then the battery can no longer inject power into the medium. Which scenario describes the actual situation remains an open problem. However, the present study seems to hint at the second hypothesis.

IV. SUMMARY

In summary, we have considered the behavior of acoustic localization in water containing many air-filled bubbles, using a simple numerical model. The localization behavior was investigated and is shown to follow the description of a simple transport equation. The research provides a fundamental backbone for a simple intuitive picture about wave localization in random media. A novel approach has been proposed for describing the localization phase transition. It is shown that a collective behavior appears when wave localization occurs. More detailed discussion of the collective behavior and a consideration of 2D situations will be published elsewhere [18].

ACKNOWLEDGMENTS

The work received support from National Science Council of ROC and also from the National Central University in the form of a CO-OP scholarship to EH. One of us (AA) also acknowledges the support from NSC and the Spanish Ministry of Education in the form of post doctoral fellowships.

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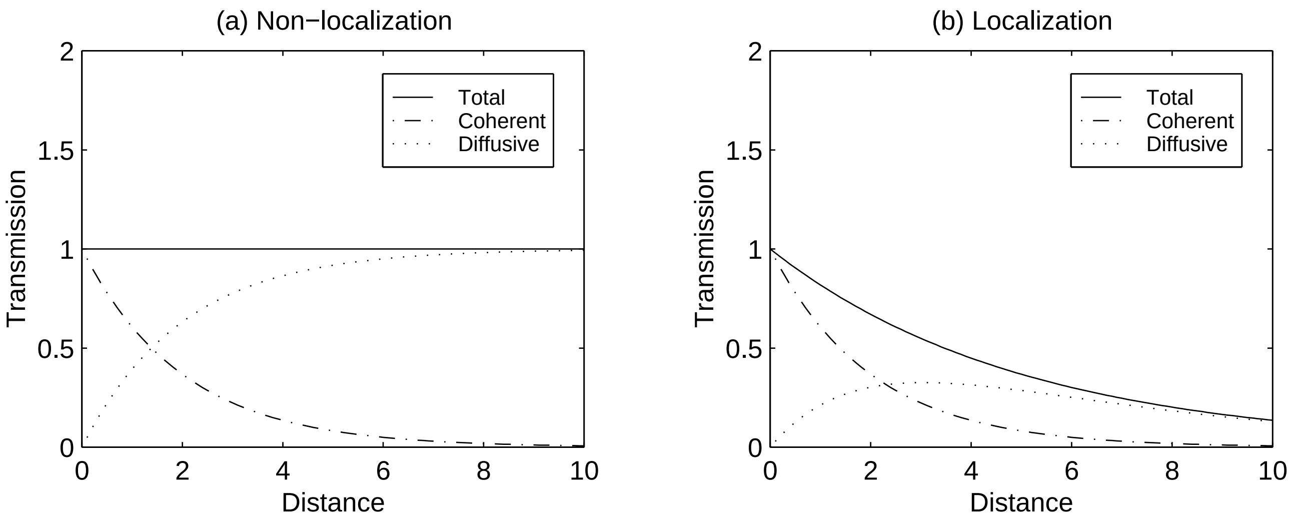
FIGURE CAPTIONS

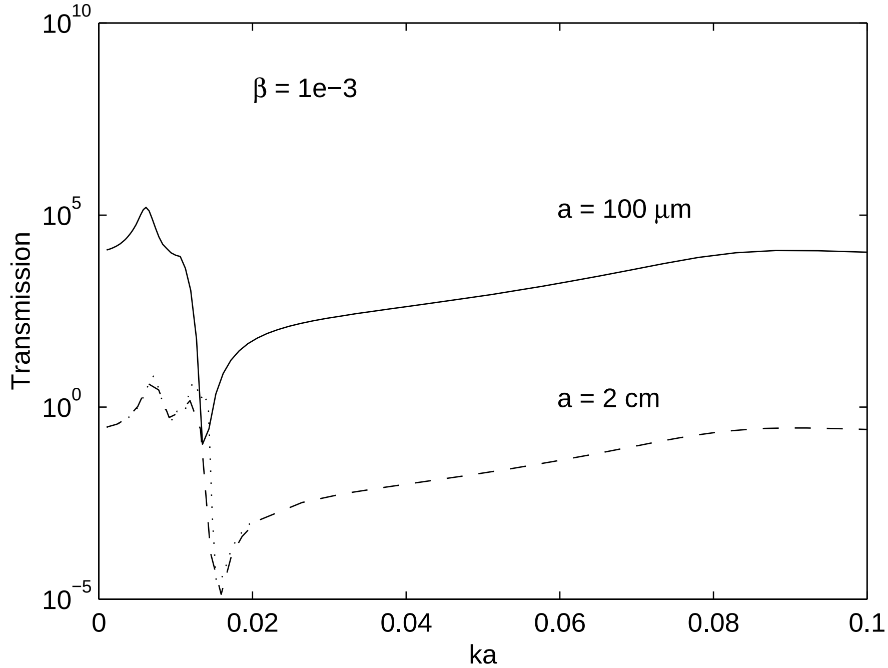
Fig. 1 A conceptual illustration of wave localization

Fig. 2 Transmission as a function of ka for two different bubble radii

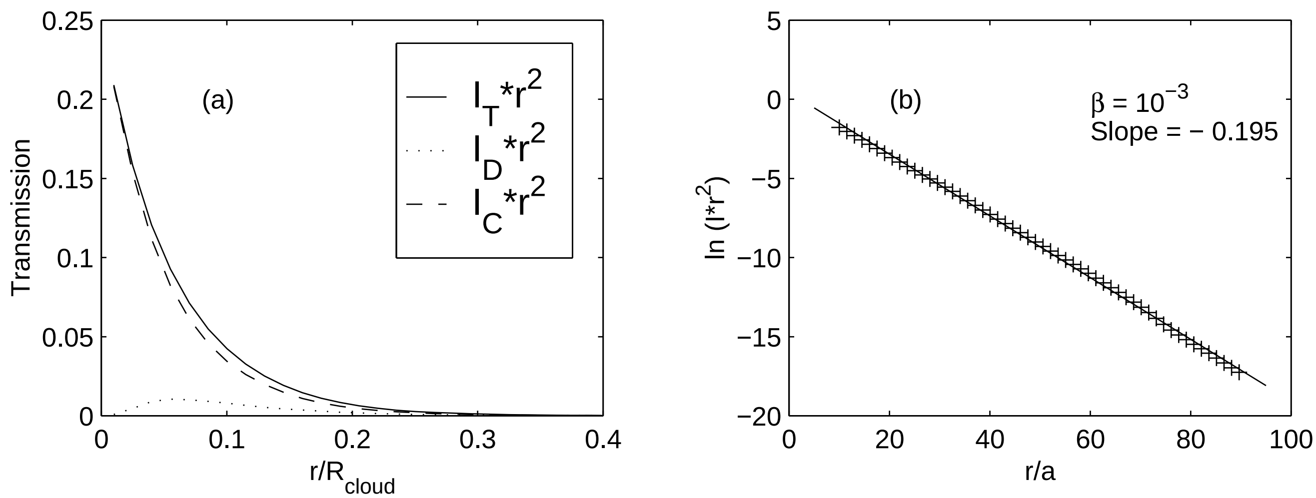
Fig. 3 Transmission as a function of propagation distance and localization length

Fig. 4 Left column: Argand diagrams for the two-dimensional phase vectors lying parallel to the x-y plane. Right column: Spatial distribution of acoustic energy (arbitrary scale).

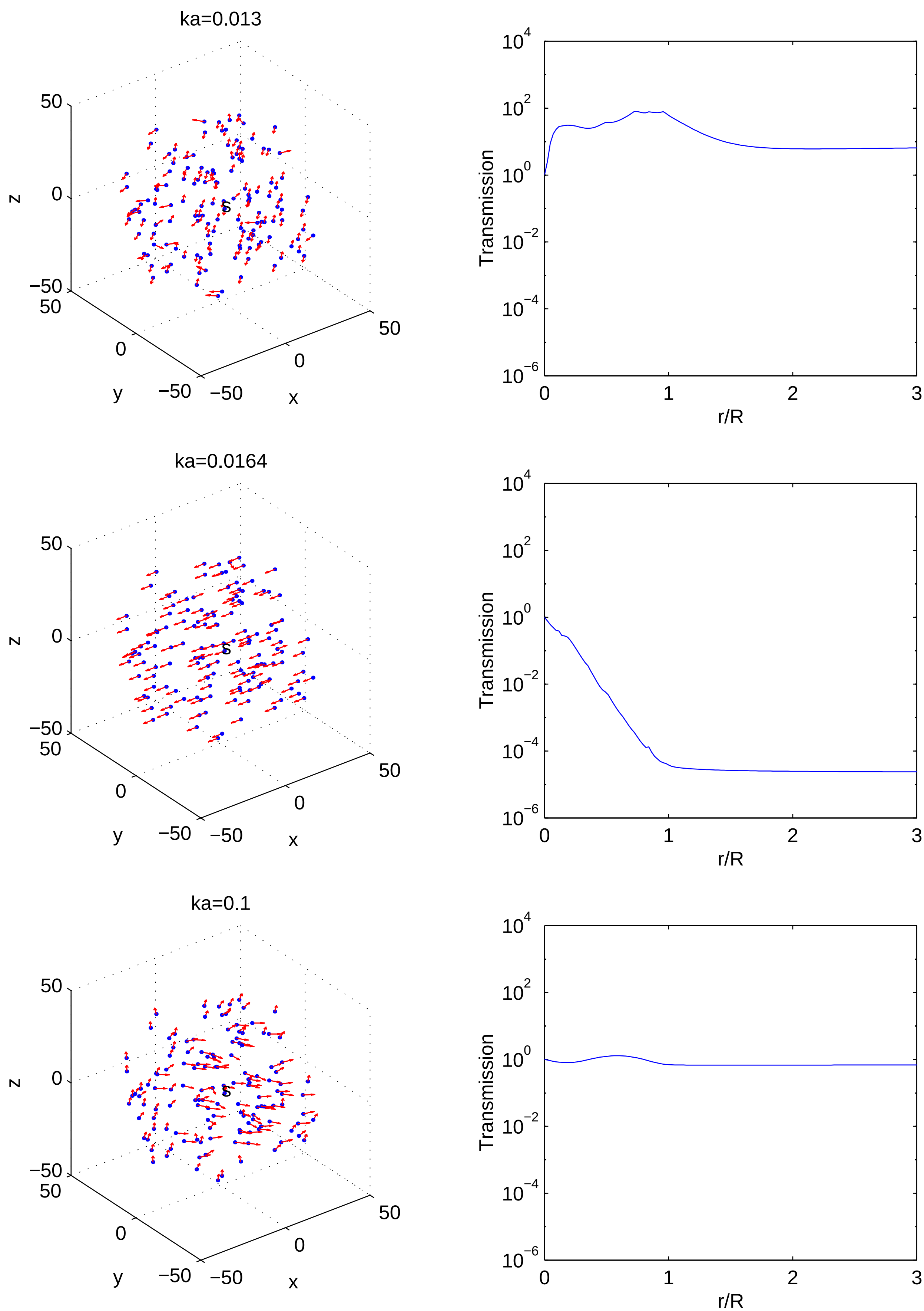

(a) Non−localization
(b) Localization
Total
Coherent
Diffusive
Transmission
Distance
0
0.5
1
1.5
2
0
2
4
6
8
10



β = 1e−3
a = 100 μm
a = 2 cm
Transmission
ka
10^10
10^5
10^0
10^−5
0
0.02
0.04
0.06
0.08
0.1



(a)
I_T*r^2
I_D*r^2
I_C*r^2
Transmission
0.25
0.2
0.15
0.1
0.05
0
0
0.1
0.2
0.3
0.4
r/R_{cloud}
(b)
$\beta = 10^{-3}$
Slope = − 0.195
$\ln (I*r^2)$
5
0
−5
−10
−15
−20
0
20
40
60
80
100
r/a


ka=0.013
ka=0.0164
ka=0.1
z
y
x
S
50
0
−50
Transmission
r/R
10^4
10^2
10^0
10^−2
10^−4
10^−6
0
1
2
3